# DETERMINING THE EFFECT OF PARTICLE COLLISION FOR A POLYFRACTIONATED MATERIAL IN A TWO-COMPONENT FLOW 

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Results are presented from the experimental determination of the freefall velocities of particles (with consideration of the collisions between these) for various flow characteristics. These data are compared with theoretical relationships derived earlier.

The quantitative relationships governing the motion and heat transfer of a polydisperse solid in a vertical two-component flow are determined to a significant degree by the collisions between particles of various dimensions. The approximate theoretical solutions of this problem (existing in the literature) have been derived on the basis of numerous simplifying assumptions [1-4]. The authors of these papers failed to take into consideration phenomena such as the lateral displacement of particles, their rotation, and their collision against the channel walls; in addition, the particle surfaces were assumed to be absolutely smooth, etc., thus resulting in the need for the introduction of substantial correction factors into the derived analytical relationships. In this connection, it is of considerable interest to undertake an experimental determination of the quantitative relationships governing the collisions of particles in a flow and the comparison of these relationships with the theoretical solutions.

A fundamental part of this problem is the determination of the quantitative relationships governing the motion of a solitary large-scale particle in a twocomponent flow; it is precisely to this phenomenon, that this present paper is devoted. The investigation was carried out on an experimental installation which is made up of a closed contour for the solid and of an open contour for the transport gas (Fig, 1). The stand consisted of glass tube $2(\mathrm{~d}=76 \mathrm{~mm}, \mathrm{~L}=9 \mathrm{~m})$, pneumatic feed 1 , settling chamber 3 , filter 4 , hopper 5 , and vacuum pump 6. The air flow rate was measured by means of a membrane, while the flow rate of the fine fraction was measured by weighing the feed mechanism at each operational stage.

Millet was used for the fine particles ( $\hat{o}_{2}=2.25 \mathrm{~mm}$, $\mathrm{V}_{2}=7.65 \mathrm{~m} / \mathrm{sec}$ ) in addition to a narrow fraction of an alumosilicate catalyst with particles approximately spherical in shape ( $\delta_{2}=2.94 \mathrm{~mm}, \mathrm{v}_{2}=9.73 \mathrm{~m} / \mathrm{sec}$ ); the large particles consisted of hollow glass beads which were filled with materials of various specific weights $\left(\delta_{1}=5.68-14.36 \mathrm{~mm}, \mathrm{~V}_{1}=16.5-29.5 \mathrm{~m} / \mathrm{sec}\right)$.

The average velocities $u_{2}$ for the fine particles at various concentrations of the materials and various gas velocities had been determined prior to the experiments, as had the values of the coefficient $k$ for the recovery of the normal velocity components of the particles on collision. For the determination of $u_{2}$ on the stabilization and acceleration segments of the mo-
tion we used an SKS-1 motion-picture camera to film the flow at a speed of $800-1500 \mathrm{frames} / \mathrm{sec}$. We car-


Fig. 1. Diagram of experimental stand.
ried out 25-40 measurements in each regime. This proved to be sufficient to prevent the widths of the confidence interval for the average-velocity value in the majority of cases from exceeding ( $0.1-0.15$ ) $\mathrm{u}_{2}$ av for a five-percent significance level.

To determine the values of the coefficients $k$ we measured the height $L_{0}$ to which the particles had risen, i.e., the particles ejected from a certain height L after collision against a horizontal plate. Since the Reynolds number for the particles assumed values in the range $\mathrm{Re}_{\mathrm{t}}=13-800$, the equation of motion for the particle has the following form [5]:

$$
\begin{equation*}
u \frac{d u}{d L}= \pm g-g\left(\frac{u}{V}\right)^{1.5} \tag{1}
\end{equation*}
$$

where the plus sign corresponds to a drop and the minus sign corresponds to a rebound. The integration of (1) yields

$$
\begin{align*}
& \frac{g}{2 a^{3}} L=-t-\frac{a}{6} \ln \frac{(a-t)^{2}}{a^{2}+a t+t^{2}}+ \\
& +\frac{a}{\sqrt{3}} \operatorname{arctg} \frac{2 t+a}{a \sqrt{3}}-\frac{\pi a \sqrt{3}}{18} \tag{2}
\end{align*}
$$

(which denotes the falling of the particle);

$$
\begin{gather*}
\frac{g}{2 a^{3}} L_{0}=t_{0}-\frac{a}{6} \ln \frac{\left(a+t_{0}\right)^{2}}{a^{2}-a t_{0}+t_{0}^{2}}- \\
-\frac{a}{\sqrt{3}} \operatorname{arctg} \frac{2 t_{0}-a}{a \sqrt{3}}-\frac{\pi a \sqrt{3}}{18} \tag{3}
\end{gather*}
$$

(which denotes the rebounding of the particle until stopped); here $t=\sqrt{u}, a=\sqrt{\mathrm{V}}$. According to the definition, the recovery factor is equal to

$$
\begin{equation*}
k=\frac{u_{0}}{u} . \tag{4}
\end{equation*}
$$

For catalyst particles whose shape is very close to the spherical, we were able to determine the value of $k$ with sufficiently high accuracy: $\mathrm{k}_{\mathrm{c}}=0.924$. For the millet (particles of elongated shape) we derived only a tentative value of $\mathrm{k}_{\mathrm{m}}$ which was equal to 0.51 .

In the first series of experiments, for various concentrations $\mu_{2}$, we determined the reduced free-fall velocity $\mathrm{v}_{1}$ for the large-scale particle (the air velocity at which the particle is carried aloft in a two-phase flow) or the minimum velocity for its entrainment. The experiments were carried out at $\operatorname{Re}=(61-137)$. $\cdot 10^{3}$ for a flow with concentrations of $\mu_{2}=0.075-3.54$ $\mathrm{kg} / \mathrm{kg}$. The results demonstrate that the particle collisions in the flow exert significant effect on particle motion, while the effect of the collisions (even with insignificant concentrations) is commensurate with other forces affecting the disperse substance (weight, aerodynamic drag). For example, a particle with a free-fall velocity $V_{1}=29.5 \mathrm{~m} / \mathrm{sec}$ at a fine-fraction concentration of $\mu_{2}=1 \mathrm{~kg} / \mathrm{kg}$ may be carried upward with a gas velocity amounting only to $17-17.5 \mathrm{~m} / \mathrm{sec}$.

These results were processed in dimensionless quantities:

$$
\begin{align*}
& y_{\mathrm{t}}=\frac{A}{g} \mu_{2} u_{2} v_{1}  \tag{5}\\
& y_{\mathrm{e}}=1-\left(v_{1} / V_{\mathrm{I}}\right)^{2} \tag{6}
\end{align*}
$$

which are easily derived from the equations of motion for the particles of the large-scale fraction [4]

$$
\begin{equation*}
g\left[\left(v_{1} / V_{1}\right)^{2}-1\right]+\alpha A \mu_{2} u_{2} v_{1}=0 \tag{7}
\end{equation*}
$$



Here we have introduced the denotation

$$
A=\frac{3}{4} \frac{(1+k) \rho_{\mathrm{g}}\left(\delta_{1}+\delta_{2}\right)^{2}}{\rho_{1} \delta_{1}^{3}+\rho_{2} \delta_{2}^{3}} .
$$

The physical significance of the dimensionless quantities follows from Eq. (7): $y_{t}$ represents the "theoretical" value of the average acceleration for the largescale particle (referred to $g$ ); this value is governed by the collisions with the fine fraction; $y_{e}$ is the experimental value of this quantity.

Since the exact value of k on collision of millet particles with a glass bead was unknown, in the calculation of $y_{t}$ it was assumed that $\mathrm{k}_{\mathrm{m}}=1$. In this connection, the correction factor $\alpha=y_{e} / y_{t}$; in addition to the above-indicated factors, we also take into consideration that the collisions are anything but absolutely elastic. We see from the experimental data (Fig. 2) that in the subject range the value of $\alpha$ is apparently independent of the particle characteristics, i.e., of the ratios $\delta_{1} / \delta_{2}$ and $\rho_{1} / \rho_{2}$ or $V_{1} / V_{2}$ (which, in the experiment, assumed values of $\delta_{1} / \delta_{2}=2.52-6.38, \rho_{1} / \rho_{2}=$ $=1.12-2.78$, and $\mathrm{v}_{1} / \mathrm{v}_{2}=1.95-3.86$ ). Appropriate statistical processing of the experimental data for the millet confirmed this assumption. In a range of small concentrations for the fine fraction ( $\mu_{2}<1-1.2 \mathrm{~kg} / \mathrm{kg}$ ) and for correspondingly small values of the parameter $y_{t}$, the coefficient $\alpha$ is a weak function of $\mu_{2}$ (or of $y_{t}$ ); for larger $y_{t}$ this coefficient diminishes markedly.

The empirical relationship between the dimensionless parameters was determined by means of the method of least squares, in the form $\alpha=a+$ by $_{t}$. Other types of correlations were less appropriate. The following relationships were obtained in the processing of the experimental data:
for the catalyst particles

$$
\begin{equation*}
\alpha=0.968-0.1974 y_{\mathrm{t}} \tag{8}
\end{equation*}
$$

for the millet

$$
\begin{equation*}
\alpha=0.8594-0.2085 y_{\mathrm{t}} \tag{9}
\end{equation*}
$$

The curves of these functions are also shown in Fig. 2. Deviation of the experimental points from the curves of (8) and (9) does not exceed $\pm 20 \%$.


Fig. 2. Relation between parameters $y_{e}$ and $y_{t}$ (series $\mathrm{I}:$ a) millet, b) catalyst): 1) $\delta_{1}=5.68 \mathrm{~mm}, \mathrm{~V}_{1}=16.5$ $\mathrm{m} / \mathrm{sec}$; 2) 7.83 and 19 ; 3) 7.45 and 25.5 ; 4) 9.87 and 29.5 ; 5) 9.72 and 19.5 ; 6) 12.1 and 19.5 ; 7) 14.36 and 22.3

In the assumption that the quantitative relationships governing the collisions are independent of the kind of fine particles, we can derive an approximate value from (8) and (9) for the millet recovery factor $\mathrm{k}_{\mathrm{m}}$. This value was determined from the minimum condition of the expression

$$
\begin{gathered}
\int_{y_{\mathrm{rmin}}}^{y_{\operatorname{mmax}}}\left[\left(0.968-0.1974 y_{\mathrm{t}}\right)-\left(0.8594 \frac{1+k_{\mathrm{c}}}{1+k_{\mathrm{m}}}-\right.\right. \\
\left.\left.\quad-0.2085 y_{\mathrm{t}}\left(\frac{1+k_{\mathrm{c}}}{1+k_{\mathrm{m}}}\right)^{2}\right)\right]^{2} d y_{\mathrm{t}}
\end{gathered}
$$

and it proved to be equal to $\mathrm{k}_{\mathrm{m}}=0.606$, which is in satisfactory agreement with the above-cited quantity $\left(\mathrm{k}_{\mathrm{m}}=0.51\right)$.

The second series of experiments involved determination of the time during which the large-scale particle passes a tube section of length $\mathrm{L}=8.96 \mathrm{~m}$, moving downward toward the ascending gas flow and thus also overcoming the impact of approaching fine particles. The experiments were carried out with the same catalyst for $\operatorname{Re}=(61-95) \cdot 10^{3}$ and concentrations $\mu_{2}=0,214-2.28 \mathrm{~kg} / \mathrm{kg}$. The time of the bead motion was measured with an electronic stopwatch.

The experimental results demonstrate that the effect of the collisions on the velocity of the descending motion for the large-scale particle was extremely significant. Thus, if the time of motion for a particle exhibiting the dimension $\delta_{1}=9.87 \mathrm{~mm}$ in pure air ( $\mathrm{w}=$ $=13.7 \mathrm{~m}$ ) is 1.52 sec , in a two-component flow, for a fine-fraction concentration of $1.65 \mathrm{~kg} / \mathrm{kg}, \tau=12.4-13$ sec.

Visual observations showed that the trajectories of motion for the large-scale particles differ little from the rectilinear. In view of this, the equation for the descending motion of the large-scale particle can be written in the form [4]

$$
\begin{gather*}
\frac{d u_{1}}{d \tau_{1}}= \\
=g\left[1-\left(\frac{w+u_{1}}{V_{1}}\right)^{2}\right]-\alpha A w \mu_{2} \frac{\left(u_{1}+u_{2}\right)^{2}}{u_{2}} . \tag{10}
\end{gather*}
$$

Equation (10) is easily integrated if we assume that $\mathrm{u}_{2}=\mathrm{const}=\overline{\mathrm{u}}_{2}$

$$
\begin{equation*}
\ln \left(\frac{r-q-p u}{r-q} \frac{r+q}{r+q+p u}\right)=-2 r \tau \tag{11}
\end{equation*}
$$

or with consideration of the fact that $u_{1} d \tau_{1}=d L$

$$
\begin{gather*}
\left(1-\frac{q}{r}\right) \ln \frac{r-q-p u}{r-q}+ \\
+\left(1+\frac{q}{r}\right) \ln \frac{r+q+p u}{r+q}=-2 p L, \tag{12}
\end{gather*}
$$

where

$$
p=\frac{q}{V_{1}^{2}}+\frac{\alpha A w \mu_{2}}{\bar{u}_{2}} ; \quad q=\frac{g w}{V_{1}^{2}}+\alpha A w \mu_{2} ;
$$

$$
r=\frac{q}{V_{1}} \sqrt{1+\frac{a A w \mu_{2}}{g \bar{u}_{2}}\left[V_{1}^{2}-\left(w-\bar{u}_{2}\right)^{2}\right]} .
$$

The value of $\bar{u}_{2}$ was determined by averaging the velocities of the fine fraction over the time of motion for the large-scale particle. Here, as shown in preliminary calculations, the error in determining $L$ does


Fig. 3. Relation between parameters ye and $y_{t}$ (series II: notation from Fig. 2).
not exceed $5 \%$. The value of $\alpha$ for each experiment was determined from Eqs. (11) and (12) on a BESM-2 computer. Figure 3 shows the results from the processing of the experimental data expressed in dimensionless quantities $y_{t}$ and $y_{e}=\alpha y_{t}$.

The structure of the parameter $y_{t}$ follows directly from (10):

$$
\begin{equation*}
y_{\mathrm{t}}=\frac{A}{g} w \mu_{2} \frac{\left(\frac{L}{\tau}+\bar{u}_{2}\right)^{2}}{\bar{u}_{2}} . \tag{13}
\end{equation*}
$$

We see from the experimental data that the nature of the relationship among $\alpha$, the particle characteristics, and the material concentration (Fig. 3) is approximately the same as in the first series of experiments. Processing by the method of least squares yields the following empirical relationship:

$$
\begin{equation*}
\alpha=0.951-0.1803 y_{t} \tag{14}
\end{equation*}
$$

which is insignificantly different from Eq. (8).
The derived experimental data thus confirm the earlier conclusion [4] as to the significant influence of the collisions on the motion of a polydisperse material in a two-component flow. However, under actual conditions, as shown by the experiments, the effect of the collisions is $10-30 \%$ smaller than in the idealized case for which the theoretical solution is valid [4]. The lateral displacements and rotation of the particles apparently play a decisive role in this case.

## NOTATION

g is the gravitational acceleration, $\mathrm{m} / \mathrm{sec}^{2} ; \mathrm{k}$ is the recovery factor for the normal velocity component; L is the flow length, m ; u is the velocity of the disperse matter, $\mathrm{m} / \mathrm{sec} ; \mathrm{u}_{0}$ is the velocity of the particle after impact, $\mathrm{m} / \mathrm{sec}$; $V$ is the free-fall velocity, $\mathrm{m} / \mathrm{sec}$; v is the reduced free-fall velocity, $\mathrm{m} / \mathrm{sec}$; w is the gas
velocity; $y_{t}$ and $y_{e}$ are dimensionless sets; $\alpha$ is the correction factor accounting for the deviation of the real from the ideal conditions of collision; $\delta$ is the particle size, $\mathrm{m} ; \mu$ is the flow rate mass concentration of material, $\mathrm{kg} / \mathrm{kg} ; \rho$ is the density, $\mathrm{kg} / \mathrm{m}^{3} ; \tau$ is the time, sec. Symbols: 1 and 2 are fractions of the polydisperse material; $g$ refers to the gas (as a subscript).

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